Application Note

Phase and Frequency Locking of Diode Lasers

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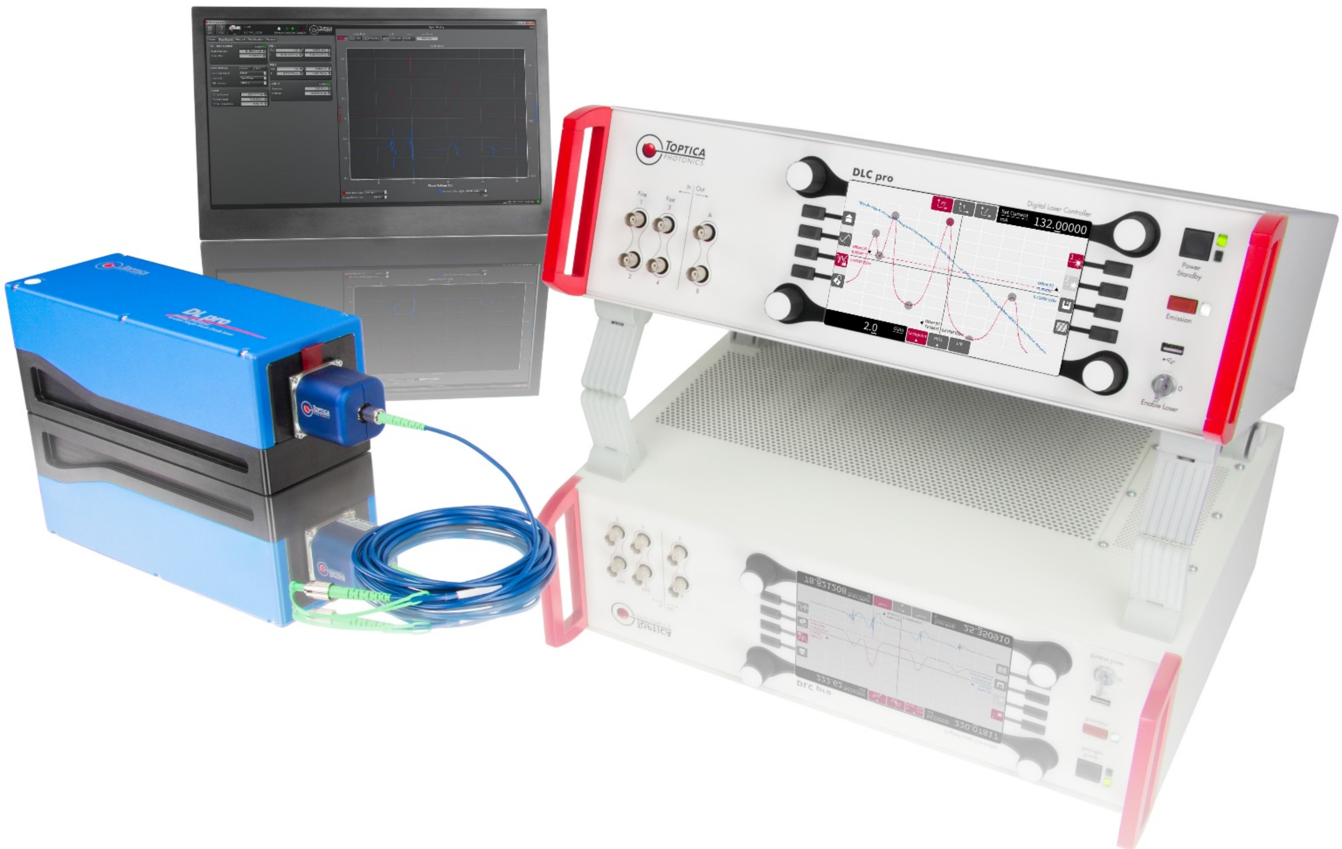

The monochromatic property of laser light is certainly one of its key features. It explains why the emergence of the laser led to a renaissance of traditional fields like atomic and molecular spectroscopy and at the same time triggered the birth of new fields like quantum optics and quantum information processing. This monochromatic property is rooted in the fundamental working principle of the laser that always contains a frequency-selective element. Examples for these elements in the case of diode lasers include external resonators and gratings written into the semiconductor.

These optical methods of frequency selection already lead to very narrow spectral features. However, a closer look reveals that frequency fluctuations occur on different time scales. Considering the diode laser, they can be of very different origin, e.g. fluctuations of the current driving the laser diode, acoustic vibrations and changes of the surrounding air pressure and temperature. To further narrow down the spectral feature, it is therefore necessary to electronically stabilize the phase or frequency of the laser to an external reference. Most generally, whenever a laser property is stabilized to an external reference, this is typically called “laser locking” or “locking”.

Many applications rely on this possibility and requirements concerning the frequency and phase stability keep increasing. While the research field of optical clocks requires extremely narrow linewidths to improve clock precision, other applications in the growing field of quantum technologies, have phase stability of a laser as a critical benchmark.

This collection of application notes provides an introduction to the principles of laser locking in general and to the more practical aspects of phase and frequency locking of diode lasers. The goal of these application notes is to provide the reader with a conceptual understanding of laser locking but also to help them choose the ideal locking solution for their application.

Generic Principle of Phase and Frequency Locking

There are many ways to lock a laser´s phase or frequency. They do, however, all follow the same basic principles based on a feedback loop, which is diagrammatically sketched in **Figure *1***. Each application note contained in this collection focuses on one element shown in this diagram.

The starting point of every feedback loop is the quantity to be stabilized. Here it is the phase or frequency of laser light. The first application note “Phase and Frequency of Laser Light” introduces some basics concerning these quantities and discusses noise sources which make laser locking necessary.

To see if the frequency changes over time, a reference is needed. Different possible references and their advantages and disadvantages are presented in the application note “Frequency References”.

The application note “Error-Signal Generation” describes how to generate an electronic signal which indicates how much the laser phase or frequency has deviated from a set value. Different schemes of how to generate that error signal are presented. Ideally at the end of this application note the reader knows which reference and which scheme of error signal generation to choose for their specific application and how to set it up.

Subsequently, the error signal is fed into a controller. It further processes the error signal and generates a control signal. Typically, a rather large number of parameters can be chosen to optimize the performance of the lock. In the application note “PID Feedback Controllers” it is discussed why a controller is needed in the first place, the specifics of a PID controller and what defines a good controller.

Finally, the control signal is fed into the so called actuator. It actually changes the frequency or phase depending on the value of the control signal. In the application note "Actuators for Laser Phase and Frequency Control" several ways to change the phase and frequency are presented together with their respective advantages and disadvantages.

To conclude, in the section "Overview of TOPTICA´s Lock Solutions", a table summarizes the considered lock solutions. It is supposed to help the reader find the most adequate lock solution for their specific application.

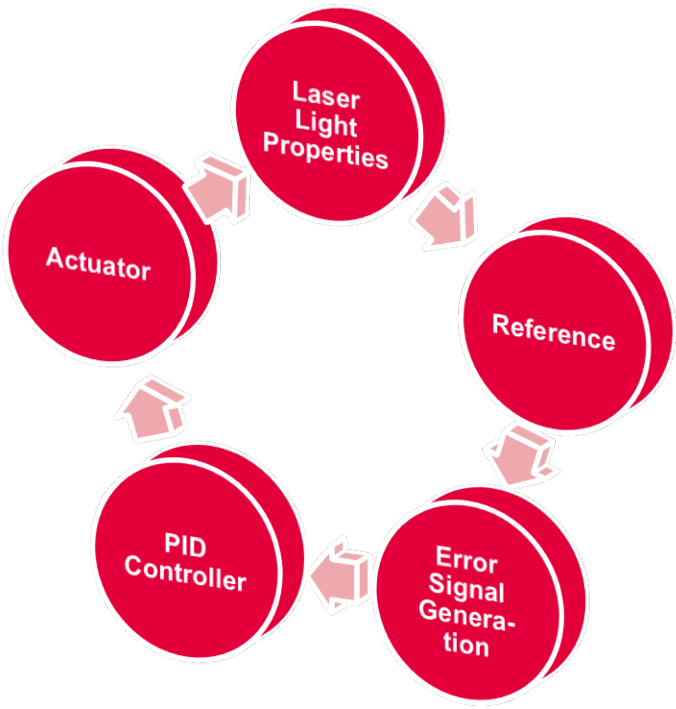

Figure 1. *Diagrammatic concept of a feedback loop for phase and frequency locking.*

Phase and Frequency of Laser Light

Laser light can be described by its space and time-dependent electric field $E(x,t)$. For the purpose of discussing phase and frequency locking of lasers, the spatial dependence is not of relevance and will be neglected. Therefore, a perfect monochromatic light field can be written as

$$E(t) = \frac{1}{2} A \cdot \exp(-2\pi i \bar{\nu} t) + c.c.,$$

where A is the amplitude and $\bar{\nu}$ the frequency of the light field. The abbreviation "c. c." stands for the complex conjugate of the preceding expression. In fact, various noise sources lead to fluctuations of the laser frequency $\nu(t) = \bar{\nu} + \tilde{\nu}(t)$ and of the amplitude $A(t)$, where $\bar{\nu}$ is the light field´s mean frequency and $\tilde{\nu}(t)$ represents time-dependent fluctuations, such that

$$E(t) = \frac{1}{2} A(t) \cdot \exp\big(-2\pi i \bar{\nu} t + \tilde{\varphi}(t)\big) + c.c.,$$

where the quantity $\tilde{\varphi}(t) = 2\pi \int_0^t \tilde{\nu}(u)\, du$ describes phase fluctuations. In this application note only frequency fluctuations $\tilde{\nu}(t)$ are considered while the amplitude $A(t)$ is assumed to be constant.

Figure *2* shows several causes for frequency fluctuations and the corresponding time scales on which they occur.

A signal which is typically considered to visualize a laser´s frequency stability is the laser spectrum $\mathcal{L}(f)$. It is the absolute square of the Fourier transform of the time-dependent electric field. The full width at half maximum (FWHM) of the spectral feature is referred to as laser linewidth and is often quoted as a measure for a laser´s frequency stability. The exact value of the linewidth not only depends on the laser itself but also on the corresponding measurement time. If the electric field is only recorded over several microseconds, considering **Figure *2***, it is clear that fluctuations like air pressure changes which occur on a time scale of several seconds do not contribute to the single-shot linewidth but rather manifest themselves as a drift or jitter of the central frequency of subsequent measurements. Typical causes for frequency drifts and jitter are changes of the environment, e.g. air pressure, humidity and temperature and changes related to the laser system like piezo drifts and imperfections of the temperature control.

Strictly speaking, the laser linewidth does not contain all information about the noise properties of a laser. A more detailed picture is offered by the power spectral density (PSD) of the phase or frequency noise, which are the absolute square of the Fourier transforms of $\tilde{\varphi}(t)$ and $\tilde{\nu}(t)$ respectively.

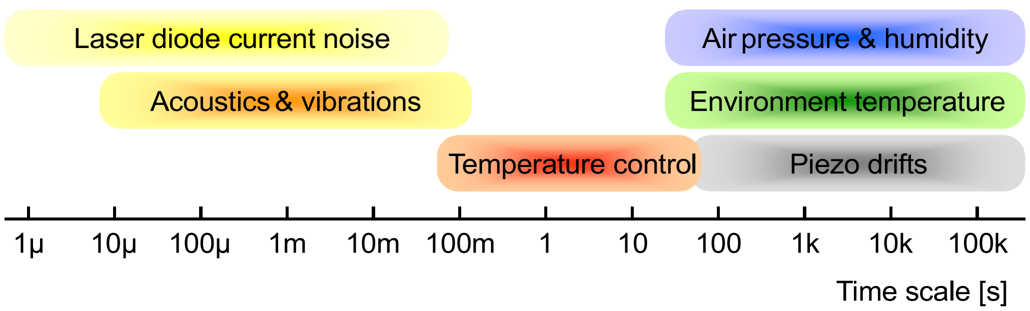

Figure 2. *Sources of laser noise.*

Frequency References

The concept of phase and frequency stabilization relies on the possibility to detect changes of the quantity that is wished to be stabilized. To do so this quantity has to be measured continuously, i.e. has to be compared to a reference. In the case of a laser´s phase or frequency, among others, this reference can be an atomic or molecular transition, an optical cavity with fixed mirror distance, another light source that is believed to be more stable, e.g. a second, stable laser or a frequency comb, or a wavelength meter. In this application note, different references are presented and their advantages and disadvantages in terms of stability and convenience are discussed.

a) Atomic or Molecular Transitions – Spectroscopy

For many applications, a good and simple way to stabilize a laser´s frequency is to compare it to an atomic or molecular transition. One advantage of this method is that optical transitions are rather narrow (typically several MHz). Furthermore, for experiments working with atoms, this method suggests itself because lasers can be locked directly to the atomic transition of interest. Therefore it is a proven method e.g. for laser cooling of atoms.

The most straightforward realization of a spectroscopy lock is to send part of the laser light through a cell which contains an atomic gas and measure the transmission with a photo diode. Light will be absorbed if the laser´s frequency is close to the resonance frequency of the atom, the closer it is the lower the transmission will be. The width of these spectral features are typically on the order of several MHz. However, the Doppler effect will lead to a broadening of these spectral features. At room temperature, e.g. for rubidium 87, this leads to a spectral width of approximately 0.3 GHz. How an error signal, which eliminates the effect of the Doppler broadening, can be generated, will be presented in the application note “Error-Signal Generation”.

TOPTICA solution

With the CoSy, TOPTICA offers a plug-and-play spectroscopy package which is based on Doppler free saturation spectroscopy. It includes the gas cell as a reference together with the optical setup and a fast photo diode for error signal generation. As the CoSy is fibre coupled no alignment is necessary.

b) Optical Cavities

An alternative approach of measuring a laser´s frequency is to link it to the geometrical properties of an optical cavity. The most common type for this purpose is the Fabry-Pérot interferometer, i.e. two parallel mirrors facing each other. The approach is based on the idea that light can only resonate, and thus be transmitted, if twice the distance between the two mirrors, i.e. the optical path length of a round-trip, is an integer multiple of the wavelength. Deviations of the laser frequency from this condition will decrease the transmission. Close to resonance the relation between transmission and frequency deviation is given by a Lorentzian function with a full-width-at-half-maximum (FWHM) linewidth

$$\Delta\nu_c = \frac{\Delta\nu_{FSR}}{\mathcal{F}},$$

where $\Delta\nu_{FSR} = \frac{c}{2l}$ is the free spectral range of the cavity, i.e. the frequency difference between adjacent resonances, c is the speed of light, l is the cavity length and $\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}$ is the Finesse[1] of the cavity. R is the intensity reflectivity of the mirrors. Therefore, to obtain small cavity linewidths, the reflectivity of the mirrors has to be large. For a cavity with a length of 10 cm and a Finesse of 100.000 this leads to a cavity linewidth of only approximately 15 kHz.

Typically, to achieve a long-term stability of the resonator length, the mirrors are mounted in a material called ultra-low expansion (ULE) glass. It has the property that its heat expansion coefficient can be brought close to zero by fine-tuning its temperature.

If it is the goal to reduce the linewidth of the laser as much as possible, a high-finesse ULE cavity certainly is the right choice. However, engineering such a cavity which fulfils the stringent requirements concerning long-term stability and finesse is a rather sophisticated undertaking. Beyond that, despite of the ULE material, on a long time scale small frequency drifts occur.

[1] $\mathcal{F}/\pi$ is the number of round-trips of a cycling light pulse before its energy has decreased to 1/e.

c) Frequency Comb

Another way to stabilize a laser´s phase or frequency is to compare it to a stable light source. A tool which has been developed for exactly this purpose is the frequency comb. Its spectrum exhibits peaks at fixed equidistant frequencies which is realized by mode-locking a spectrally broad laser. In the time domain this corresponds to a regular train of laser pulses with repetition rate f_{rep}, which equals the separation between the peaks of the comb spectrum. The relative stability between individual peaks is obtained by locking f_{rep} to a radio frequency (RF) reference. To obtain absolute stability, in addition, either the offset frequency f_{CEO}, called carrier-envelope offset, of the periodic pattern from zero has to be stabilized to a reference or a method called difference frequency generation, which passively sets $f_{CEO} = 0$, has to be implemented[2]. Therefore the long-term stability of the individual peaks is inherited from the underlying RF reference. It is therefore possible to transfer the long-term stability of e.g. an atomic clock to the regime of optical frequencies. Such a degree of stability cannot be offered by any other optical reference.

The frequency comb is the perfect tool if many lasers are to be stabilized to a common reference. Locking different lasers to a common reference is a particularly good idea if relative fluctuations between different lasers are of importance. Using a frequency comb, relative stability can be achieved between lasers separated in wavelength by several hundreds of nm. Additionally stabilizing a frequency comb to a short-term stable optical reference[3] results in the best laser reference system available.

TOPTICA solution

The TOPTICA DFC CORE + is a frequency comb which is based on Difference Frequency Generation (DFG), i.e. the carrier-envelop offset is passively fixed to $f_{CEO} = 0$. It is therefore inherently stable and combines high robustness and high-end performance. Several wavelength extensions are available to bridge the wavelength range between 420 and 2200 nm. On top of that the DFC core + can be extended by further hardware for beat detection and processing. Together with fast locking electronics and a convenient control software this forms a complete reference system for controlling and locking several lasers.

d) A Second, Stable Laser

A laser locked to one of the above mentioned references can serve as a reference itself. Difference frequencies are limited by the bandwidth of the employed photodetector, typically up to several GHz, unless a frequency comb is employed. With this method it is possible to achieve near-perfect relative phase stability between the two lasers, which is advantageous for many applications. A typical application example is a phase stable Raman pair addressing two hyperfine levels.

[2] A more detailed description of a frequency comb can be found here.
[3] As an optical reference, typically, a laser locked to a high-finesse cavity is used.

e) Wavelength Meter

A very convenient way to determine the frequency of a laser is to use a wavelength meter. With typical measurement ranges covering more than an octave, they offer the possibility to lock the laser to almost any frequency. However, the accuracy of a wavelength meter is typically limited to several MHz. Also, the time needed for measuring the wavelength and generating a control signal is much larger than for other schemes. Wavelength meters are therefore mostly used for stabilizing the laser against drifts occurring on time scales of seconds and more but not for reducing the laser´s linewidth.

Error-Signal Generation

This application note deals with the question of how to generate an error signal for phase and frequency stabilization of lasers. First, general schemes which are independent of the choice of reference are discussed. After that, schemes based on the references discussed in the application note "Frequency References" are presented.

The goal of error signal generation is to create an electronic signal that is proportional to the laser´s deviation from a certain set frequency or phase which will be called lock point in the following. **Figure *3*** shows what such an error signal could look like. If the laser frequency is too small, the signal is negative. If its frequency is too large, the signal is positive and it is zero at the lock point. A requirement for a good frequency stabilization is a large signal-to-noise ratio, i.e. small frequency deviations should cause a large signal. Considering **Figure *3*** this means that the steepness of the slope around the lock point is desired to be large.

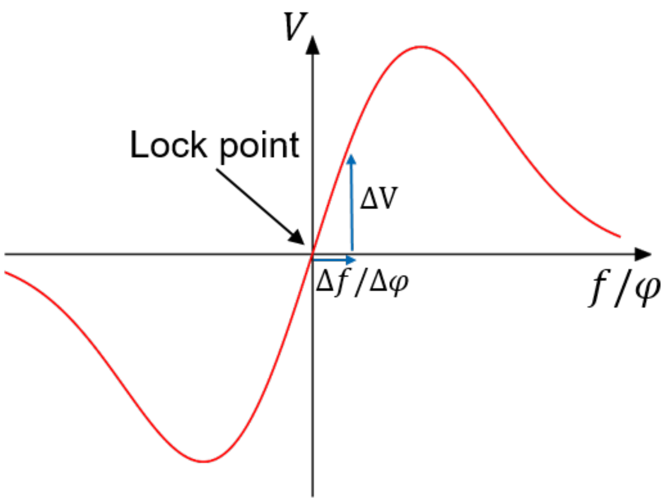

Figure 3. *Sketch of a typical error signal. The lock point is given by the zero-crossing. The steepness of the slope around the zero-crossing affects the sensitivity of the signal to phase or frequency fluctuations. Ideally, the error signal would be a purely linear function of phase or frequency. However, the error signal shown here is more realistic as it is given by the derivative of a spectral peak or dip.*

In this application note, it is discriminated between locking the phase and locking the frequency. In principle the result is equivalent because if the phase offset is constant over time, then the frequency offset will be as well. However, laser locking based on an error signal which is a function of a laser´s frequency deviation will be referred to as frequency locking while laser locking based on an error signal which is a function of a laser´s phase deviation will be referred to as phase locking. The difference will become clear in the following examples.

Quite generally, generating an error signal consists of two steps. The first step is to create an optical signal using an optical setup involving a reference, e.g. one of those presented in the application note "Frequency References". This is necessary because visible light exhibits frequencies around several hundreds of THz which makes it impossible to directly convert this into an electronic signal with the same frequency. Therefore, using the reference, the optical signal is converted to lower frequencies which are directly detectable. Consequently, the second step is to record the optical signal using e.g. a photo-diode or a camera and subsequently to further process this signal electronically.

General Error-Signal-Generation Schemes

a) Side-of-fringe locking

Considering a spectral peak, e.g. obtained from the transmission through a spectroscopy cell or through an optical resonator, a straight forward way to obtain an error signal is to add or subtract a constant offset. **Figure *4*** illustrates this scheme. Then there will be two zero crossings, one at the left and the other at the right side of the peak. In the vicinity e.g. of the left zero crossing, the signal will be positive if the frequency is larger than the frequency at the zero crossing and the signal will be negative if the frequency is smaller than the frequency at the zero crossing. Consequently this can be used as an error signal for stabilizing the laser to the frequency at the zero crossing. Because the laser will be locked to the side of the peak, this locking scheme is called side-of-fringe locking. A problem of this scheme is that amplitude fluctuations will cause the position of the zero crossing to fluctuate, i.e. the lock will convert amplitude noise of the laser into frequency noise which is undesirable. It is important to note that side-of-fringe locking is naturally limited to frequencies above or below the spectral feature. It is a natural consequence of this particular locking scheme that locking the laser exactly on the spectral feature is impossible.

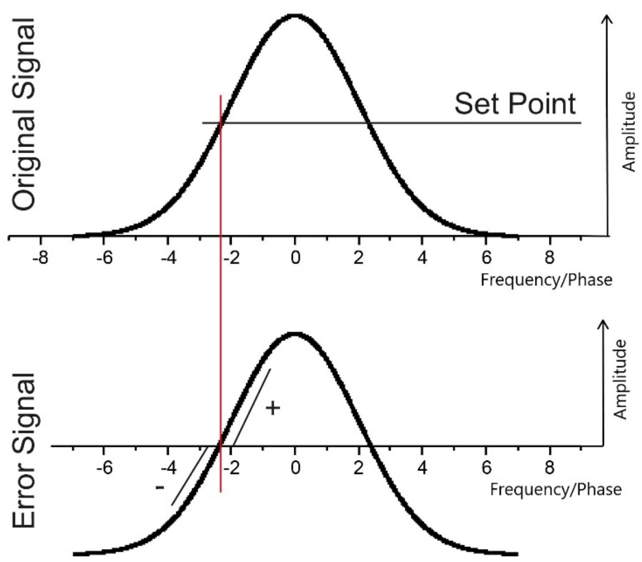

Figure 4. *By adding or subtracting a constant value, an error signal can be obtained from a spectral peak or dip.*

b) FM spectroscopy and top-of-fringe locking

An alternative method is based on the principle of frequency modulation (FM) spectroscopy. The laser frequency is modulated, e.g. by modulating the laser current or with an EOM. As the frequency is scanned over a spectral feature, the frequency modulation will be converted into an amplitude modulation (AM). Assuming modulation amplitudes smaller than the width of the spectral feature, the amplitude of this signal will be proportional to the slope of the spectral feature at the centre frequency of the modulation. This relation is shown in the upper plot of **Figure *5***. Using the lock-in detection principle, this AM signal can be demodulated by mixing it with the radio frequency (RF) used for modulating the laser frequency or the phase of an EOM. This yields a DC signal whose dependence on laser frequency is shown in the lower plot of **Figure *5***. It is proportional to the slope of the spectral feature. It has all desirable features of an error signal as described earlier and can be used to lock the laser frequency to the top of the peak. Therefore, this scheme is also referred to as top-of-fringe locking.

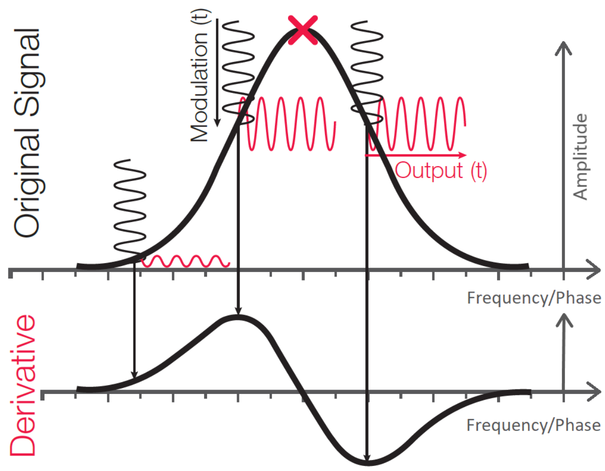

Figure 5. *Top-of-fringe principle. Modulating the laser frequency and demodulating the PD signal with the same frequency leads to an error signal which is proportional to the derivative of the original signal.*

Atomic Spectroscopy Lock

A straightforward optical setup for realizing a spectroscopy lock was already discussed in the application note "Frequency References". However, to get rid of the Doppler broadening, a different setup is needed and sketched in **Figure *6***. A beam derived from the laser to be stabilized is split into two parts using a beam splitter (BS). Most of the light is transmitted and is called the pump beam. Only a small fraction (typically around 10%) of the light is reflected and is called probe beam. The beams are guided such that probe and pump beam are counter-propagating.[4] Behind the gas cell the intensity of the probe beam is recorded with a photo diode (PD). If the pump beam is blocked and the laser frequency is scanned, i.e. changed over time, then the Doppler broadened peak shown in **Figure *7*** (a) is recorded. If the pump beam is on, however, narrow peaks on top of the Doppler broadened profile occur, shown in **Figure *7*** (b), which correspond to the Hyperfine levels of the atoms. This is because, as mentioned above, the velocity of the atoms is governed by probability and

[4] In some realizations a telescope is used to widen the pump beam such that the probe beam has large overlap with the pump beam inside the gas cell.

therefore a fraction of the atoms happen to have almost zero velocity. Therefore, if the laser hits the resonance of a Hyperfine level the strong pump beam saturates the atomic gas, i.e. the atoms are not able to absorb further photons. Therefore the probe beam will be transmitted. If the laser frequency is off-resonant, e.g. detuned to a lower frequency, only atoms will be excited which counter-propagate the laser light with a velocity matching the Doppler shift. As probe and pump beam counter-propagate, both lasers excite atoms which move in opposite directions. Therefore, the group of atoms that is excited by the probe beam is not the same group of atoms that is in resonance with the pump beam. Consequently, the transmission of the probe beam is unchanged by the presence of the pump beam if the laser frequency is not on resonance with the unshifted atomic transition.

The Doppler broadened transmission of a second probe beam which is sent through the gas cell without overlapping the pump beam can be used as a reference. Subtracting this reference from the actual signal yields the signal shown in **Figure *7*** (c).

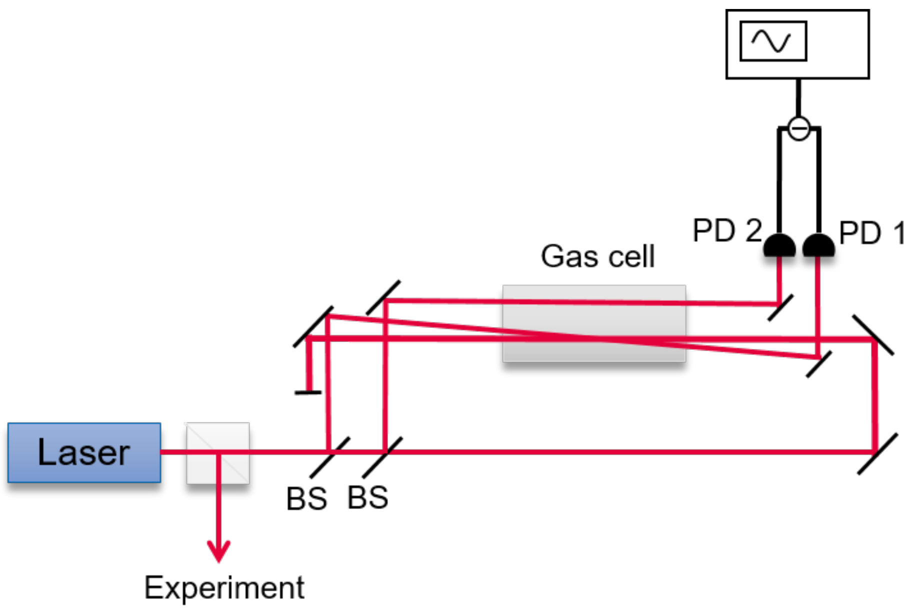

Figure 6. *Optical setup for Doppler-free saturation spectroscopy. A strong pump beam is counter-propagating the weak probe beam. If the laser hits a resonance, pump and probe beam address the same group of atoms that have zero velocity along the beam axes. The strong pump beam saturates those atoms such that probe light absorption is minimal. This leads to high transmission of the probe beam.*

TOPTICA solution

TOPTICA´s CoSy includes a complete optical setup for Doppler-free saturation spectroscopy together with a controller which performs the subtraction of the reference signal from the spectroscopy signal. The laser light is simply fed to the CoSy via a fibre, so no alignment is necessary. The subtracted signal is available at one of the output ports of the controller.

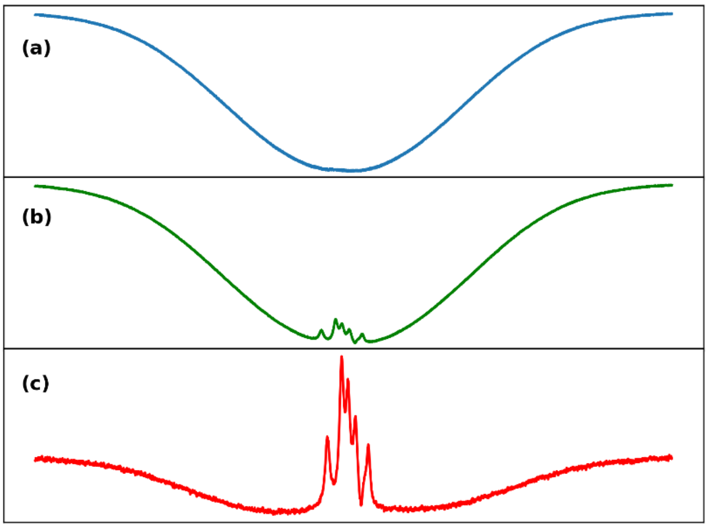

Figure 7. *(a) Doppler-broadened spectrum; (b) Spectrum obtained from Doppler-free saturation spectroscopy; (c) Doppler-free spectrum reduced by Doppler broadened spectrum*

Figure 8. *The CoSy together with its controller is a convenient way to realize a frequency lock, e.g. for laser cooling.*

Considering one hyperfine peak of this spectroscopy signal, an error signal can be generated following one of the methods described in the sections "Side-of-fringe locking" and "FM spectroscopy and top-of-fringe locking". In both options the error signal indicates how much the laser frequency deviates from a set point. Therefore we refer to locks which are based on these error signals as frequency locks.

TOPTICA solution

Both options of generating the error signal from the spectroscopy signal are supported by TOPTICA´s DLC pro Lock, which is a software option of TOPTICA´s laser controller DLC pro and will be further discussed in the application note "PID Feedback Controllers". It offers the possibility to choose between side-of-fringe and top-of-fringe locking. In the case of side-of-fringe locking it internally performs the subtraction of an adjustable constant value. In the case of top-of-fringe locking it performs the modulation of the laser current. Alternatively, it provides a signal which can be used to generate the phase or frequency modulation using e.g. an electro-optic modulator (EOM). Beyond that, it also performs the demodulation of the spectroscopy signal to yield the error signal shown in *Figure 5*. Together with the CoSy, the DLC pro Lock integrated in the DLC pro offers one of the most convenient ways of error signal generation. This combination is all you need to stabilize the frequency of TOPTICA´s diode lasers. The clear and simple setup needed for realizing an atomic spectroscopy lock is sketched in *Figure 9*.

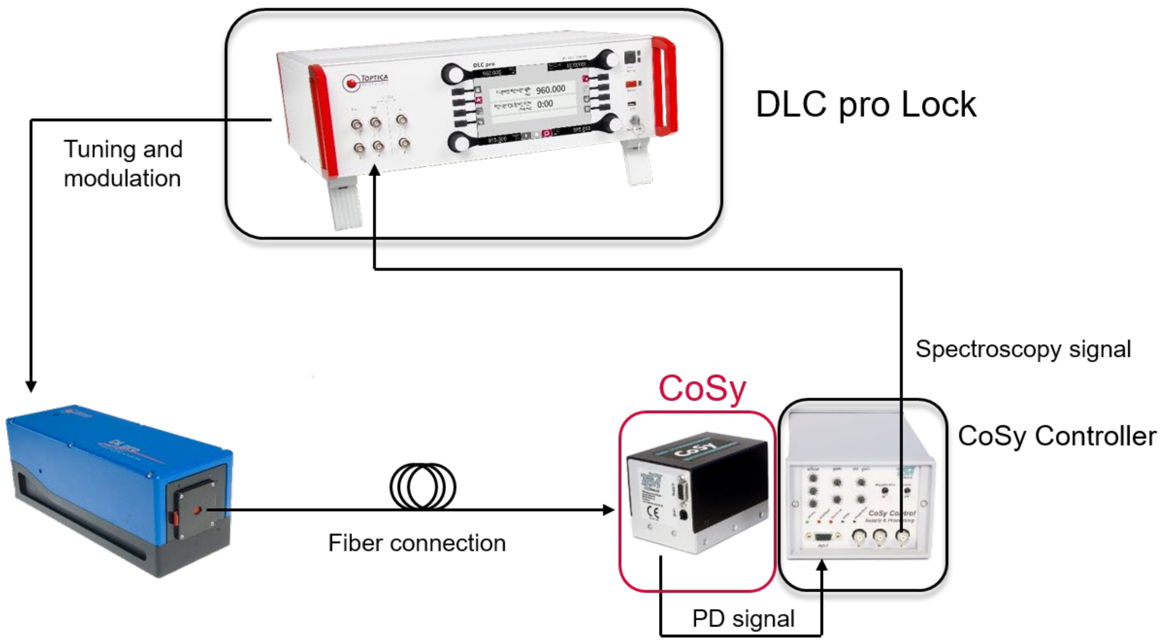

Figure 9. *Schematic of a spectroscopy lock using TOPTICA components.*

Locking to a Reference Resonator

While the width of the spectral feature in atomic spectroscopy is ultimately limited by the natural linewidth of the specific transition, the spectral feature of a Fabry-Pérot cavity is given by the reflectivity of its mirrors and the distance between them. Cavity linewidths of several kHz can be reached. This makes error signals with a very steep slope and therefore with a very good signal-to-noise ratio possible.

The spectral feature can be obtained from the light either transmitted through or reflected from the cavity. A typical cavity transmission is shown in **Figure *10***.

The most straightforward way to generate an error signal is to send part of the laser light through the cavity and measure the transmission with a photodiode behind the cavity. Analogous to the techniques described before, side-of-fringe and top-of-fringe locking can be realized using this transmission signal. However, fast fluctuations of the laser phase or frequency are spectrally represented by frequency components with a large deviation from the laser´s central frequency[5]. If this deviation is larger than the cavity linewidth, then these components will be suppressed in the cavity transmission, i.e. the cavity acts as a low pass filter for phase and frequency noise. Therefore a lock built upon an error signal obtained from the cavity transmission will have low performance suppressing noise components at frequencies larger than the cavity linewidth. We therefore focus on schemes that use the reflection of the cavity.

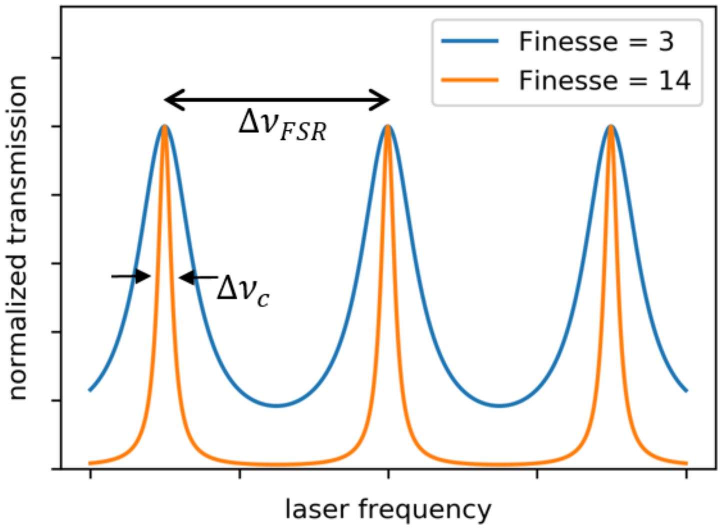

Figure 10. *Transmission of a Fabry-Pérot resonator. High reflectivity of the mirrors (orange) leads to a small spectral width $\Delta\nu_c$. The distance between cavity resonances $\Delta\nu_{FSR}$ is determined by the length of the resonator.*

The implementation of side-of-fringe locking using the cavity reflection is straightforward, but has the disadvantages described above. Instead, we will focus on a scheme known as the Pound-Drever-Hall (PDH) technique. It is based on FM spectroscopy and uses the reflection of a cavity. The optical setup used for the generation of the PDH error signal is shown in **Figure 11**. Part of the laser light, which is separated from the main beam using a non-polarizing beam splitter (NPBS), is sent through an EOM which modulates the phase of the field with a modulation frequency Ω typically much larger than the cavity linewidth. This modulation leads to frequency sidebands separated by Ω from the central frequency (carrier). The reflected light is separated from the incident light using the combination of a polarizing beam splitter (PBS) and a λ/4 waveplate. Because the frequency sidebands are off-resonant, they will be reflected from the first cavity mirror without acquiring a phase shift, while the carrier enters the cavity and obtains a phase shift that depends on the deviation of the laser frequency from the cavity resonance. The superposition of both components creates a beat signal which oscillates with frequency Ω and whose phase depends on the frequency of the laser. Subsequently, this signal is recorded with a photodiode. By mixing the signal from the photodiode with a reference RF signal with the same frequency Ω, a DC signal which is proportional

[5] This is true as long as the amplitude of phase modulations is much smaller than one and the amplitude of frequency modulations is much smaller than the modulation frequency.

to the phase of the PD signal, and thus depends on the frequency of the laser light, is obtained. It is important to adjust the phase between the reference signal and the PD signal to obtain the maximum amplitude of the mixed signal. The signal obtained from this procedure is depicted in **Figure *12***. It has a zero crossing on resonance and thus serves as an error signal for top-of-fringe locking. The steepness of the slope around resonance is inversely proportional to the cavity linewidth, i.e. a small cavity linewidth yields large signal-to-noise ratio. A more detailed introduction to the PDH method can be found in the great article by Eric D. Black[6]. As the PDH error signal is a function of the frequency deviation, locks based on it are frequency locks.

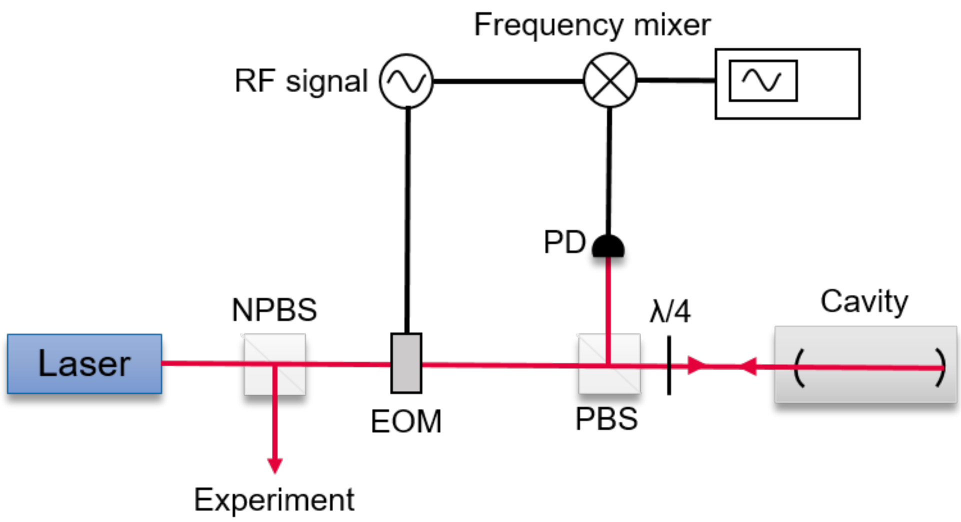

Figure 11. *Schematic setup for the Pound-Drever-Hall technique. Optical paths are depicted in red, electronic signal paths in black.*

TOPTICA solution

With the PDH/DLC pro, TOPTICA offers a module for the electronic generation of a PDH signal. This includes an RF signal e.g. for driving the EOM. The PD signal recording the reflection of the cavity is fed into the input of the PDH/DLC pro module. Parameters like phase shift and gains can comfortably be adjusted using the DLC pro interface. As PID controllers, both the DLC pro Lock as well as external controllers like the FALC pro can be employed. While the DLC pro Lock is typically used to transfer the long-term stability of e.g. a ULE cavity to the laser, the FALC pro can actually be used to dramatically decrease the laser linewidth. A similar setup based on TOPTICA lasers and locking electronics was used to realize linewidths of below 1 Hz. It is recommended to extend the DLC pro Lock with the PDH/DLC pro if it is important to avoid modulation sidebands in the kHz range.

[6] Eric D. Black, *Am. J. Phys.* **69**, 79 (2001)

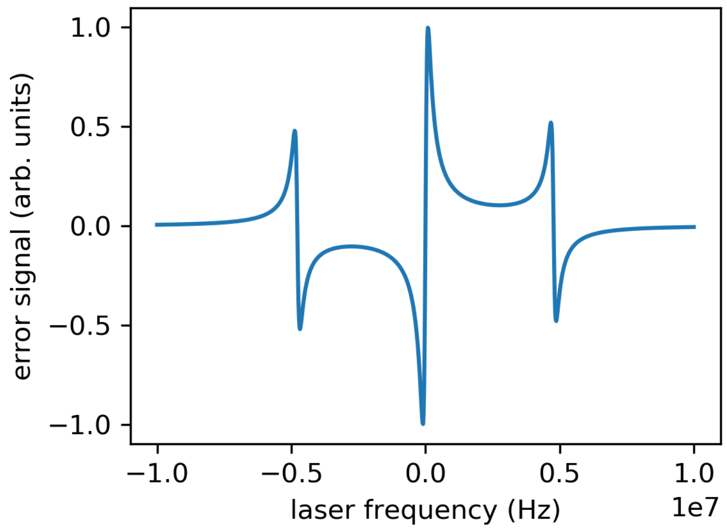

Figure 12. *PDH error signal. It features a steep slope around the lock point at 0 Hz and a large capture range determined by the modulation frequency of 5 MHz. The second and third zero crossing at -5 and +5 MHz correspond to one sideband being resonant with the cavity and interfering with the reflected carrier and other sideband.*

Phase Lock to an Optical Reference

There are various ways to lock a laser to another optical reference. Here, the general principle of a phase lock to an optical reference will be described. The optical reference can e.g. be a second cw laser or a frequency comb. This method allows to establish coherence between the laser and the optical reference and between the lasers which are locked to the same optical reference. By establishing a phase lock to a frequency comb, coherence between lasers separated by several hundreds of nm can be realized.

Figure *13* shows the sketch of an optical setup for the implementation of an offset phase lock. Light from the optical reference is superimposed with the laser light using a beam splitter (BS), for convenience this can also be a fibre-based beam splitter. Polarization has to be taken care of such that ideally polarizations of the comb light and of the laser light are identical. The superposition of both beams is recorded with a photodiode and gives rise to a beat signal at the difference frequency between the laser light and the optical reference. In the case of a frequency comb, the optical signal will contain beats between the laser light and many comb peaks. However, the set value for the laser frequency is typically chosen close to one peak such that beats with other peaks at higher frequencies can be filtered easily.

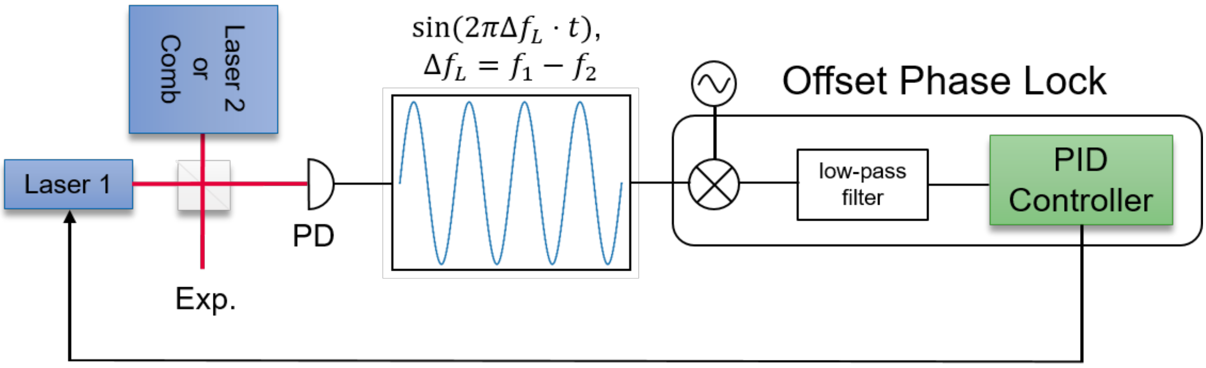

Figure 13. *Optical setup for the implementation of an offset phase lock.*

The PD signal is a function of the phase difference between the laser light and the optical reference under consideration. This signal is only constant if the frequency difference between the laser light and the optical reference is exactly zero. Assuming that at time t=0 this value is zero, any phase deviation will lead to a deviation of the signal from zero. It can therefore directly be used as an error signal for phase stabilization.

One disadvantage of this method is that DC noise, e.g. coming from the photodiode, is converted into frequency noise. Instead the PD signal is mixed with a reference RF signal with frequency Ω. The signal obtained from this procedure will only be constant if the frequency difference between the laser light and the optical reference is exactly Ω. The error signal is a function of the phase difference between the laser light and the optical reference apart from the constant frequency offset. Therefore, it is a phase lock.

An offset phase lock to a second cw laser can easily be implemented using TOPTICA´s FALC pro which will be presented in more detail in the application note "PID Feedback Controllers". For implementing an offset phase lock, the FALC pro includes a fast PID controller and a frequency mixer which combines the electronics after the PD shown in **Figure *13***.

TOPTICA solution

TOPTICA offers a complete solution for locking a laser to a TOPTICA frequency comb. The DFC BC is an advanced beam combiner module. It offers the possibility to adjust the power ratio of the beams and it features a pure cw-light output. The DFC MD is a monochromatic detector for detecting the beat signal. It contains a grating-based adjustable filter to get rid of all unwanted frequency components of the beat signal. Beyond that, it features a low-noise photo detector. Error signal generation is performed with TOPTICA´s PFD. It is a phase- frequency detector and includes a tunable RF source. This package is completed by a convenient software interface which integrates all components and allows controlling the lock to the comb in an easy and straightforward way.

PID Feedback Controllers

This application note discusses the purpose and principles of controllers by considering the action of a PID controller as a prototypical example on the transfer function of a feedback loop. Beyond that TOPTICA´s PID controllers will be discussed regarding specifications and typical applications.

The purpose of a controller can be understood in the context of general control theory. This is a rather old field and a plethora of text books have been written which fully cover this topic, e.g. the book by R. C. Dorf and R. H. Bishop [7]gives a good overview for a physicist´s practical point of view. Therefore, this application note only gives a short summary of important aspects relevant for the topic of controllers. **Figure *14*** shows a closed loop diagram typically considered in control theory. It is related to **Figure *1*** which summarizes the general concept of phase and frequency locking of lasers. $r(t), e(t), u(t)$ and $y(t)$ are the time-dependent reference, error signal, control signal and output signal, respectively. $d(t)$ describes noise which perturbs the system.[8] The time-independent quantities $C(s), P(s)$ and $F(s)$ denote the action of the controller, the plant (in this series of application note it is called "***Actuators***") and the sensor, respectively. In this series of application notes $F(s)$ includes the optical setup and the opto-electronic sensor described in the application note "***Error-Signal Generation***". Analysing this system using the Laplace transforms[9] $R(s), E(s), U(s), Y(s)$ and $D(s)$ of the variables $r(t), e(t), u(t), y(t)$ and $d(t)$, yields the relation

$$Y(s) = \left(\frac{P(s)C(s)}{1 + P(s)C(s)F(s)}\right) R(s) + \left(\frac{1}{1 + P(s)C(s)F(s)}\right) D(s) = T(s)R(s) + S(s)D(s)$$

between the reference signal, the noise entering the system and the output signal . $T(s) = \frac{P(s)C(s)}{1+P(s)C(s)F(s)}$ is called the reference transfer function of the feedback loop, $S(s) = \left(\frac{1}{1+P(s)C(s)F(s)}\right)$ is the disturbance transfer function, while $C(s)$ is the transfer function of the controller. An example of $C(s)$ is shown in **Figure *15***. From this relation two important statements can be derived. First, for a large gain *C(s)* of the controller and $|F(s)| \approx 1$ the output signal closely follows the reference signal and disturbances are suppressed effectively. Second, if the overall phase of the product *P(s)C(s)F(s)* (phase delay) reaches 90°, then the magnitude of the denominator $1 + P(s)C(s)F(s)$ can become very small and the feedback loop starts amplifying deviations from the reference value instead of damping them. From these statements two key requirements for the controller can be formulated. First, the parameters of the controller should be chosen such that the phase of the product *P(s)C(s)F(s)* has a value between -90° and +90° up to a frequency s_{bw} as large as possible. Second, the magnitude of *the product P(s)C(s)F(s)*, i.e. the gain of the feedback loop*,* should be much larger than 1 for values of *s* smaller than the loop bandwidth s_{bw}, while it should be much smaller than 1 for frequencies *s* larger than the loop bandwidth.

[7] R. C. Dorf and R. H. Bishop, *Modern Control Systems* (Pearson, 2016).

[8] Here the perturbation enters the system at the output of the feedback loop. In general, there could also be a disturbance which enters the system between the controller *C* and the plant *P*. The distinction between both types of perturbation will not be of importance for the conceptual discussion of a controller.

[9] The variable *s* is related to the frequency *f* following the relation $s = 2\pi i \cdot f$.

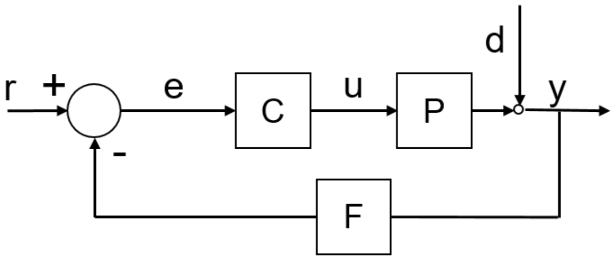

Figure 14. *Diagram of a closed loop system. The reference signal r(t) is compared to the output signal y(t) which is measured with a sensor F. The difference between these signals is the error signal e(t) which is processed by the controller C and generates the control signal u(t). The control signal is fed into the plant P which generates the output signal y(t). d(t) describes noise which perturbs the system.*

The PID controller is the prototype for most controllers, where the P, I and D represent different parts of the controller which contribute to the control signal *u(t)*.The overall signal is the sum of these three parts. The P stands for the proportional part and contributes to the control signal *u(t)* a term which is proportional to the error signal $K_p \cdot e(t)$. The I stands for the integral part and contributes a term which is proportional to the integral of the error signal $K_I \int_0^t d\tau\, e(\tau)$. The D stands for the derivative part of the controller and contributes a term which is proportional to the derivative of the error signal $K_D \cdot \frac{de(t)}{dt}$. The control signal *u(t)* is given by the sum of these three parts

$$u(t) = K_p \cdot e(t) + K_I \int_0^t d\tau\, e(\tau) + K_D \cdot \frac{de(t)}{dt}.$$

Applying the Laplace transformation to this equation yields the controller transfer function

$$C(s) = K_P + \frac{K_I}{s} + K_D s.$$

While K_I can be used to adjust the transfer function for lower frequencies, K_D has an impact on the high frequency part of the transfer function. K_P acts on the intermediate frequency range. Beyond that, considering the relation $s = 2\pi i \cdot f$, this equation shows that the integral part induces a phase shift of -90°, while the derivative part introduces a phase shift of +90°.

Most controllers are a combination of these three elements. However, there are several ways how to combine and physically implement these elements. Consequently, there is a large variety of controllers with different features available. But what defines a good controller? This depends on the requirements of the application. In the following, several criteria relevant for laser locking will be discussed.

One of the most important benchmarks is the bandwidth of the controller defined above. Intuitively it can be understood as the rate at which deviations from the set value can be corrected. If the laser linewidth is of critical importance, a controller with a bandwidth as high as possible should be chosen. The bandwidth of the controller should be understood as an upper limit of the possibly achievable bandwidth of the feedback loop. Typically, the actual bandwidth of the feedback loop will be significantly smaller. Related to the bandwidth is the signal delay of the controller which describes how much time it takes for a signal to be transferred from the input to the output of the controller.

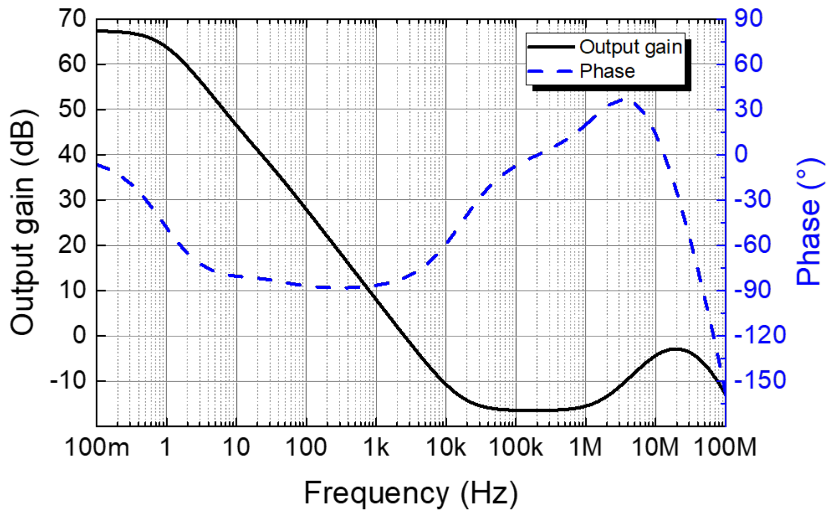

Figure 15. *Example of a transfer function as obtained with TOPTICA´s fast PID regulator FALC 110. The amplitude of the transfer function (black) describes the frequency-dependent gain of the controller. The phase of the transfer function (blue) describes the phase difference between the input and the output signal.*

Another important criterion is convenience. Many applications require remote control of laser systems. To realize this, controllers with a digital interface are typically preferred over purely analog ones because in many cases they feature a software interface which allows remote control. Beyond that, more sophisticated features like automatic relocking and signal analysis can be implemented more easily.

TOPTICA offers a variety of controllers which address different requirements.

Table *1* gives an overview of TOPTICA controllers and their characterization in terms of bandwidth and convenience.

The DLC pro lock is a digital lock based on a field-programmable gate array (FPGA) and reaches bandwidths of up to 30 kHz. It excels in terms of convenience and ease of use as it provides remote locking and features like automatic relock. The so-called click&lock function lets you easily choose your set point by just clicking on it on the touchscreen of the DLC pro. It offers the possibilities of side-of-fringe and top-of-fringe locking and is thus an excellent choice for realizing a spectroscopy lock. On top of that it includes a feature which automatically optimizes the lock using an algorithm which analyses the response of the feedback loop and thus finds the optimal PID parameters.

The FALC pro is a high-performance controller with unrivalled bandwidth and signal delay. It is the perfect choice for reducing the laser linewidth, e.g. by realizing a PDH lock on a high-finesse cavity in combination with the PDH/DLC pro module. While it is an analog controller and thus reaches extremely low signal delays of typically 10 ns, it features a digital interface, which is responsible for its very convenient usability. It allows remote locking and combining it with the DLC pro lock enables the automatic relock and the click&lock function. It also includes a frequency mixer and thus is the perfect choice to implement an offset phase lock. The FALC pro is the successor of the FALC 110 and the mFALC 110.

Table 1. *Overview of controllers offered by TOPTICA.*

	FALC pro	DLC pro Lock	DigiLock 110
Description	High-performance solution for linewidth reduction; digital interface (remote locking)	Digital controller (integrated in DLC pro); High convenience (Relock, Click&Lock)	Versatile digital locking solution + analysis features
Bandwidth	50 MHz (-3dB)	≈ 30 kHz	≈ 10 MHz (analog)
Signal delay (typ.)	10 ns	10 µs	200 ns (digital)

The DigiLock 110 is the combination of a digital and an analog controller which makes a high bandwidth of up to 10 MHz possible while offering various analysis features. With the parameters given, the DigiLock can calculate the controller frequency response in amplitude and phase and visualize it, which is a useful tool in optimizing locking performance. Beyond that the DigiLock offers network analysis functions for determining actuator bandwidth and resonance frequencies.

Actuators for Laser Phase and Frequency Control

The last element of every feedback loop for phase and frequency locking is the actuator. It performs phase and frequency adjustments based on the control signal. While external components like electro-optic modulators (EOMs) or acousto-optic modulators (AOMs) can be used to change the frequency or phase, in this application note we focus on laser parameters that can be adjusted and lead to frequency changes.

A laser type which is predestined for fast and broadband tuning is the external cavity diode laser (ECDL). Here we will discuss four ways to adjust the frequency of an ECDL: changing the temperature of the diode, changing the piezo voltage which controls position and angle of the grating, adjusting the current which drives the laser diode, and changing the voltage of an intra-cavity EOM.

Changing the temperature of the diode will change the laser frequency by a large amount, typical tuning coefficients are changes of 0.1 nm/K. However, as it typically takes quite a while for the diode to thermalize (several seconds for a change of 1 K) this is a very slow way to adjust the frequency and therefore this option is rarely used for this purpose. However, for another type of diode lasers based on DFB diodes (Distributed FeedBack diodes) without an external grating, changing the temperature is indeed used to adjust the laser frequency.

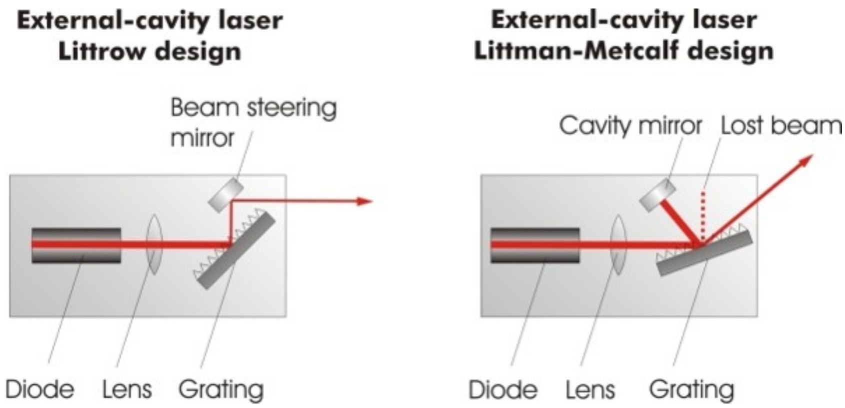

Figure 16. *Different configurations of external cavity diode lasers (ECDL).*

In TOPTICA´s ECDLs the grating used for frequency selection is mounted on a piezo. Changing the piezo voltage changes the position and the angle of the grating and thus leads to a frequency change. It offers the possibility to adjust the frequency over a large range. Typical tuning coefficients are on the order of 1 GHz/V. Locking the laser frequency via the piezo voltage offers a bandwidth of several kHz and is therefore typically used to counteract rather slow frequency drifts.

Modulating the current driving the laser diode changes the refractive index of the semiconductor and thus also leads to frequency changes. Typical tuning coefficients are approximately 0.1 GHz/mA. Modulating the current already allows to modulate the laser frequency with a bandwidth of several MHz. It is therefore the typical actuator used for reducing the laser linewidth. Typically a current lock is combined with a slow piezo lock counteracting large drifts because changing the laser current also causes changes of the laser power. Therefore, only fast and small frequency fluctuations in the kHz range should be compensated by changing the laser current.

A method which is not so common yet, is an intra-cavity EOM which also allows to quickly change the refractive index. It allows for even faster frequency adjustments while the range over which the frequency can be tuned is typically very small.

Overview of TOPTICA´s Lock Solutions

Table *2* shows a summary of lock solutions discussed in this series of application notes and of TOPTICA laser locking electronics that can be used to realize these locks. The first column lists the different types of locking schemes that are discussed in this series of application notes. The second column gives typical application examples for each locking scheme. The third column lists TOPTICA devices which are designed to generate an error signal for individual locking schemes while the fourth column suggests the corresponding TOPTICA PID controller.

Table 2. *Overview of locks discussed in this article and TOPTICA components necessary to set up these locks.*

	Application	Error Signal Gerneration	Controller
Spectroscopy Lock	Laser cooling	CoSy/DLC pro Lock	DLC pro Lock
Cavity Lock	Linewidth reduction	PDH/DLC pro	FALC pro
Phase Lock to Laser	Phase stable Raman pair	FALC pro	FALC pro
Phase Lock to Comb	Long term stable lasers	DFC Core + BC+MD+PFD	FALC pro
Wavelength Meter	Drift compensation	HighFinesse Wavelength Meter	PC Software